

Inference at \*  
of proof for Lemma fun\_exp\_add-sq:

$\vdash \forall n, m: \mathbb{N}, f, x: \text{Top}. (f^{n+m}(x)) \sim (f^n(f^m(x)))$   
by ((Unfold 'fun\_exp' ( 0)·)  
CollapseTHEN (((InductionOnNat)  
CollapseTHEN (((  
Reduce 0)  
CollapseTHEN (((((UnivCD)  
CollapseTHEN (Auto·))·)  
CollapseTHEN ((Try (  
(Complete (Auto·))·))·))·))·))·

1:

1.  $n : \mathbb{Z}$
  2.  $0 < n$
  3.  $\forall m: \mathbb{N}, f, x: \text{Top}.$   
 $(\text{primrec}((n - 1) + m; \lambda x.x; \lambda i.g. f \circ g)(x))$   
 $\sim$   
 $(\text{primrec}(n - 1; \lambda x.x; \lambda i.g. f \circ g)(\text{primrec}(m; \lambda x.x; \lambda i.g. f \circ g)(x)))$
  4.  $m : \mathbb{N}$
  5.  $f : \text{Top}$
  6.  $x : \text{Top}$
- $\vdash (\text{primrec}(n+m; \lambda x.x; \lambda i.g. f \circ g)(x))$   
 $\sim$   
 $(\text{primrec}(n; \lambda x.x; \lambda i.g. f \circ g)(\text{primrec}(m; \lambda x.x; \lambda i.g. f \circ g)(x)))$